Code No: 152AA **R18**

JAWAHARLAL NEHRU TECHNOLOGICAL UNIVERSITY HYDERABAD
B. Tech I Year II Semester Examinations, September - 2023
MATHEMATICS - II
(Common to CE, ME, ECE, EIE, MCT, MMT, ECM, AE, MIE, PTM, CSE(AI&ML), CSE(IOT), AI&DS, AI&ML)

Time: 3 Hours **Max. Marks: 75**

Note: i) Question paper consists of Part A, Part B.
ii) Part A is compulsory, which carries 25 marks. In Part A, Answer all questions.
iii) In Part B, Answer any one question from each unit. Each question carries 10 marks and may have a, b as sub questions.

PART – A

(25 Marks)

1.a) Reduce the equation $y^2\ (y - xp) = x^4p^2$ to Clairaut's form. [2]

b) Solve $x\frac{dy}{dx} + y = y^2 \log x$. [3]

c) Solve $(D^3 - 8)y = 0$, where $D = d/dx$. [2]

d) Solve $(x^3D^3 + 3x^2D^2 - 2xD + 2)\,y = 0$, where $D = d/dx$. [3]

e) Which double integral gives the area of a plate in the form of a quadrant of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$? [2]

f) Find the equivalent double integral of $I = \int_0^{\frac{\pi}{2}} \int_x^{\frac{\pi}{2}} \frac{\sin y}{y} dydx$ by changing the order of integration. [3]

g) If $\bar{A}$ and $\bar{B}$ are solenoidal, then find curl$(\bar{A} \times \bar{B})$. [2]

h) If $\vec{V} = \frac{x\bar{i}+y\bar{j}+z\bar{k}}{\sqrt{x^2+y^2+z^2}}$, find the value of curl $\vec{V}$. [3]

i) State Stoke's theorem. [2]

j) If $\vec{F} = 2z\bar{i} - x\bar{j} + y\bar{k}$, evaluate $\iiint_V \vec{F}\, dv$, where v is the region bounded by the surfaces: $x = 0,\ y = 0,\ x = 2,\ y = 4,\ z = x^2,\ z = 2$. [3]

PART - B

(50 Marks)

2.a) A body cools in air of constant temperature $T_m = 20°C$. If the temperature of the body changes from 100°C to 60°C in 20 minutes, determine how much more time it will need for the temperature to fall to 30°C?

b) Uranium disintegrates at a rate proportional to the amount then present at any instant. If M_1 and M_2 grams of uranium are present at times T_1 and T_2 respectively, find the half-life of uranium. [5+5]

OR

3.a) Solve $(3y - 2xy^3)dx + (4x - 3x^2y^2)dy = 0$.

b) A pipe 20 cm in diameter contains steam at 150°C and is protected with a covering 5 cm thick for which k = 0.0025. If the temperature of the outer surface of the covering is 40°C, find the temperature half-way through the covering under steady state conditions. [5+5]

4. Solve $(D^4 + D^2 + 1)y = e^{-\frac{x}{2}} \cos\frac{x\sqrt{3}}{2}$. [10]

OR

5. Solve $(D^3 + 1)y = e^{2x} \sin x + e^{\frac{x}{2}} \sin\frac{\sqrt{3}\,x}{2}$. [10]

6. Write the triple integral in Cartesian Coordinates which gives the volume of the sphere $x^2 + y^2 + z^2 = a^2$. Evaluate that triple integral by converting it into spherical polar coordinates. [10]

OR

7.a) Find the volume cut off from the paraboloid $x^2 + \frac{y^2}{4} + z = 1$ by the plane $z = 0$.

b) A cylindrical hole of radius b is bored through a sphere of radius a. Find the volume of the remaining solid. [5+5]

8.a) Prove that $\vec{F} = r^2\vec{r}$ is conservative and find the scalar potential ϕ such that $\vec{F} = \nabla\phi$.

b) A vector field is given by $\vec{A} = (x^2 + xy^2)\bar{i} + (y^2 + x^2y)\bar{j}$. Show that the field is irrotational and find the scalar potential. [5+5]

OR

9.a) Suppose a vector field is given by $\vec{F} = (x^2 - y^2 + x)\bar{i} - (2xy + y)\bar{j}$. Is this field irrotational? If so, find its scalar potential.

b) Show that $\vec{V}(x, y, z) = 2xyz\bar{i} + (x^2z + 2y)\bar{j} + x^2y\bar{k}$ is irrotational and find a scalar function $u(x, y, z)$ such that $\vec{V} = grad\,(u)$. [5+5]

10. State and Verify the Gauss divergence theorem for $\vec{F} = (x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ taken over the rectangular parallelopiped $0 \le x \le a; 0 \le y \le b; 0 \le z \le c$. [10]

OR

11. State Green's theorem in the plane using this theorem find, $\oint(3x^2 - 8y^2)dx + (4y - 6xy)dy$ where C is the boundary of the region bounded by $x \ge 0,\ y \le 0$ and $2x - 3y = 6$. [10]

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